

A Mixed-Integer Linear Programming Model for the Multi-Trip Multi-Period Vehicle Routing Problem with Hard Time Windows

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ABSTRACT

Distributors that serve many customers with strict delivery time windows over several days must decide which vehicle serves each customer, on which trip, and on which day. This study formulates a mixed-integer linear programming (MILP) model for the multi-trip multi-period vehicle routing problem with hard time windows (MTMPVRPTW) by integrating multi-trip and multi-period formulations to minimize transportation cost under vehicle-capacity, route-duration, and time-window constraints. The model was validated conceptually against the operations of a snack distributor in Padang, Indonesia, verified through dimensional analysis, and tested in LINGO on three reduced instances with five, eight, and ten customers served by two 50-bale vehicles. It generated one route in one day, three routes including a second trip within one day, and four routes over two days, respectively, with every route within capacity. Computation time rose from 2 s to 30 min 11 s, indicating that larger instances require heuristic solution approaches.

Keywords

Mixed-integer linear programming; Multi-trip routing; Multi-period routing; Hard time windows; Vehicle routing problem

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INTRODUCTION

Distribution delivers products from suppliers to customers and is therefore a core function of the supply chain [1]. Because it directly shapes supply chain costs, distribution performance is a key determinant of company profitability [2]. Products that remain at the factory create no value for customers, and their movement to customers depends on vehicles; the planning of delivery routes is therefore one of the most direct means of controlling distribution cost [3].

Transportation is the largest cost element within this process. It has been reported to account for approximately 40%–50% of total logistics costs [4], so reducing transportation costs can indirectly increase profit [1]. One effective way to achieve such reductions is to optimize the distribution system and use existing transportation resources more efficiently [5]. Route planning lies at the center of this strategy because it determines the sequence in which customers are visited on tours that start and end at a central depot [6].

Route planning becomes difficult when products are shipped from a single origin to many destinations. A large number of delivery points requires complex routing decisions, which lengthen travel distances and times and, in turn, raise transportation costs. Such decisions are formalized as the vehicle routing problem (VRP), in which each vehicle has a limited capacity. The problem becomes considerably harder when each customer also imposes a delivery time window, giving the vehicle routing problem with time windows (VRPTW), because routes must then satisfy both vehicle capacity and customer-specific time constraints. Time windows may be soft, so that late service is permitted at the cost of penalties such as customer fines or additional transportation time [7], or hard, so that no violation is permitted and no penalty term is needed. Additional transportation time may force a distributor to postpone deliveries to the next day and thus extend the overall distribution process. In the VRPTW, a vehicle that arrives before a customer's time window opens incurs waiting time, whereas a late arrival is deferred to the next available window [8], and Zhang et al. [9] showed that VRPTW-based routing can reduce both waiting time and the number of vehicles required.

These difficulties are present in the distribution system that motivates this study. A distributor in Padang City, Indonesia, delivers its products to customers spread across the city, and each customer has its own demand and time window. Distribution extends over more than one day, and the number of trips that each vehicle may perform is not limited, so the company must determine vehicle routes for several days at once. The problem therefore combines two VRPTW extensions simultaneously: the multi-trip VRPTW (MTVRPTW), in which a vehicle may perform several trips within a working day, and the multi-period VRPTW (MPVRPTW), in which deliveries are planned over several days.

The multi-trip extension has been studied extensively. Like other VRP variants, the MTVRPTW is NP-hard, and its difficulty is increased by the need to schedule several trips for each vehicle [10], [11] and to sequence every trip so that it complies with the customers' time windows. Vehicles may perform several trips within a specified maximum working duration, and time windows additionally require each trip to start within a designated interval [10], [12]. Multi-trip routing is widely used in urban logistics and short-distance delivery [13]–[15]; Pan et al. [13], for example, used an MTVRPTW model to plan and evaluate the speed and duration of urban transportation routes, and multi-trip routing has also been applied to emergency material delivery after disasters [16]. Formulations that combine multiple trips with delivery options or multiple time windows have been used to avoid costly detours in parcel delivery and in the distribution of fresh agricultural products, thereby lowering transportation costs and accelerating delivery [17], [18]. The benefit of multiple trips, however, still depends on how close the customers are to the depot [17].

Whereas the multi-trip extension divides a vehicle's working day into several tours, the multi-period extension distributes deliveries across days. In the MPVRPTW, vehicles serve customers according to their requests, so that not every customer needs to be visited every day [19], and trips subject to time windows can be spread over several days. Keskin et al. [20] addressed multi-period routing with orders that arrive dynamically and therefore require daily information updates, whereas Fazi et al. [21] applied multi-trip routing to truck management in container drayage while accounting for the re-use of empty containers. Multi-trip routing has also been combined with multiple depots for post-disaster emergency response [22]. A survey of vehicle routing problems over time places periodic routing, routing with release dates, and multi-trip routing in a common class of time-extended problems [23]. Within this broader family, multi-depot, multi-product routing has been combined with green vehicle routing to reduce carbon emissions [24], and multi-period VRPTW models have been applied to humanitarian and military logistics, where they can also accommodate disruptions when a vehicle is unable to continue its trip within a period [25].

The two formulations closest to the present problem each capture only one of these dimensions. Gökçe and Ercan [26] formulated a multi-period VRPTW for routing relief deliveries to neighborhood disaster stations over several days, using heterogeneous vehicles and allowing stations to be visited more than once. Hernandez et al. [27] developed an MTVRPTW with limited duration (MTVRPTW-LD), in which a single vehicle performs multiple trips and the limited duration represents the maximum working time of drivers, and solved it with an exact algorithm. Neither model represents a distributor that operates homogeneous vehicles, serves each customer exactly once, allows an unrestricted number of trips per day, and plans these trips over several days under hard time windows. Route-planning studies of Indonesian distribution and transportation cases, which have applied the savings matrix method [3], the Clarke–Wright savings algorithm [28], and cuckoo search [29], have likewise focused on constructing efficient routes rather than on scheduling multiple trips jointly over several days. A formulation that integrates multi-trip and multi-period decisions under hard time windows for this type of distributor is therefore still lacking.

To address this gap, this study adapts the multi-period concept of Gökçe and Ercan [26] and the multi-trip concept of Hernandez et al. [27] and integrates them into a single mixed-integer linear programming (MILP) model, termed the multi-trip multi-period vehicle routing problem with time windows (MTMPVRPTW). The model allows one vehicle to perform several routes in a day over a planning horizon of several days and minimizes transportation cost subject to vehicle capacity, maximum delivery time, and hard customer time windows. The objective of the study is to formulate this model and to determine, through conceptual validation, unit verification, and testing on reduced instances derived from the Padang distributor, whether it generates logically correct single-trip, multi-trip, and multi-period routing solutions. The study therefore examines the correctness and computational behavior of the formulation rather than the cost savings of a full-scale implementation.

METHOD

The study followed a model-development design in five stages, summarized in Figure 1: adaptation of existing multi-period and multi-trip VRPTW models, formulation of the MTMPVRPTW as an MILP, conceptual validation against the distributor's operations, unit verification of every function, and logical testing on three reduced scenarios. The formulation is presented first, followed by the procedures used to validate, verify, and test it.

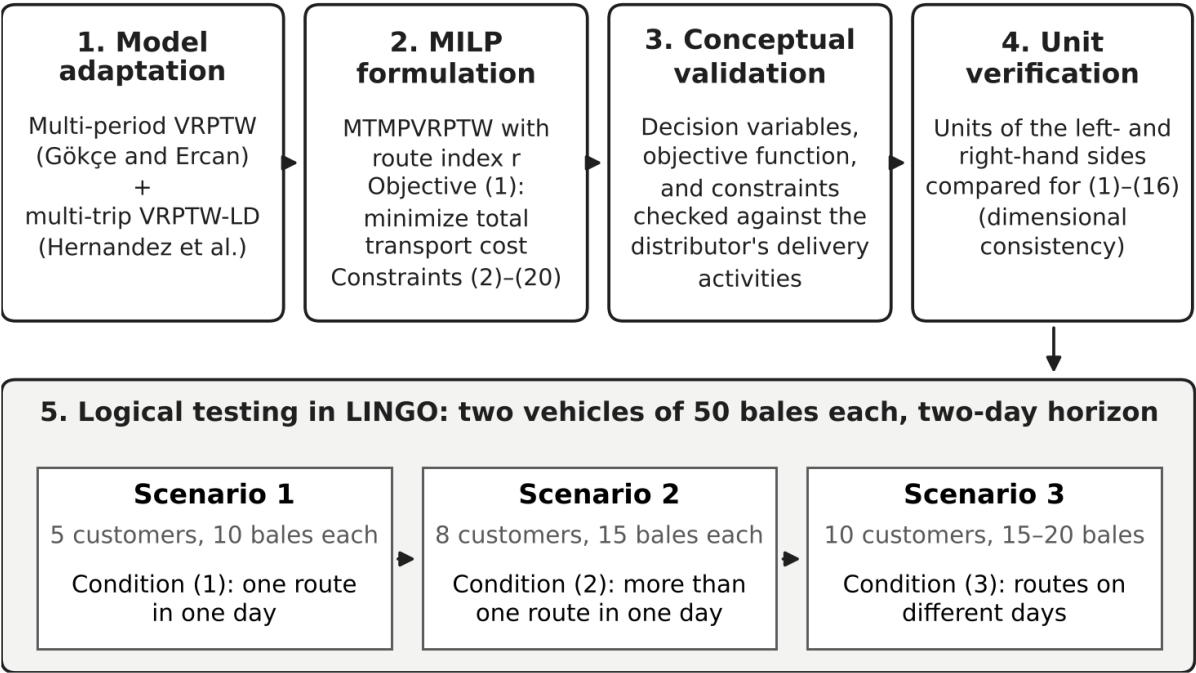


Figure 1. Research workflow for developing, validating, verifying, and testing the MTMPVRPTW model.

Model Formulation

The MTMPVRPTW is a VRPTW whose time windows cannot be violated; consequently, no penalty cost is introduced into the objective function. Its structure was adapted from the multi-period VRPTW model [26] and the multi-trip VRPTW model [27], and a route index r was added to represent the successive trips that a vehicle performs within a day, so that the formulation belongs to the class of vehicle routing problems over time in which routing decisions are allocated to both trips and periods [23]. The model considers a single depot, denoted as node 0, a homogeneous fleet, and customers that are each served exactly once within the planning horizon. The sets, parameters, and decision variables that define the formulation are listed in Table 1.

Table 1. Sets, parameters, and decision variables of the MTMPVRPTW model

Symbol	Description
Sets	
K	Set of vehicles, $k = 1, \dots, K$
H	Set of days, $h = 1, \dots, H$
N	Set of customers, $n = 1, \dots, N$; node 0 denotes the depot
R	Set of routes, $r = 1, \dots, R$
Parameters	
c_{ij}	Cost incurred on the journey from customer i to customer j
t_{ij}	Travel time from customer i to customer j
d_i	Demand of customer i
q	Vehicle capacity
a_i	Earliest arrival time at customer i
b_i	Latest arrival time at customer i
s_i	Service time at customer i
Jk	Maximum delivery time
$tstart_r$	Delivery start time of route r
$tback_r$	Final delivery time of route r
M	Sufficiently large positive constant
Decision variables	
$x_{ijk rh}$	Binary variable equal to 1 if vehicle k travels from customer i to customer j on route r on day h , and 0 otherwise
$y_{ik rh}$	Binary variable equal to 1 if customer i is visited by vehicle k on route r on day h , and 0 otherwise
z_{kruh}	Binary variable equal to 1 if vehicle k performs route u after route r on day h , and 0 otherwise
$\sigma_{k rh}$	Binary variable equal to 1 if route r is performed by vehicle k on day h , and 0 otherwise
$w_{ik rh}$	Arrival time at customer i by vehicle k on route r on day h

Using this notation, the objective function (1) minimizes the total transportation cost over all arcs, vehicles, days, and routes:

$$\text{minimize } \sum_{i=0}^N \sum_{j=0, j \neq i}^N \sum_{k=1}^K \sum_{h=1}^H \sum_{r=1}^R c_{ij} x_{ijk rh} \quad (1)$$

The objective is subject to the following constraints:

$$\sum_{j=1}^N x_{0jkrh} \leq 1 \quad \forall h \in H, k \in K, r \in R \quad (2)$$

$$\sum_{j=1, j \neq i}^N x_{ijk rh} = \sum_{j=1, j \neq i}^N x_{jik rh} \quad \forall i \in N, k \in K, h \in H, r \in R \quad (3)$$

$$\sum_{j=1, j \neq i}^N x_{ijk rh} = y_{ik rh} \quad \forall i \in N, k \in K, h \in H, r \in R \quad (4)$$

$$\sum_{j=1, j \neq i}^N x_{jik rh} = y_{ik rh} \quad \forall i \in N, k \in K, h \in H, r \in R \quad (5)$$

$$\sum_{h=1}^H \sum_{k=1}^K \sum_{r=1}^R y_{ik rh} = 1 \quad \forall i \in N \quad (6)$$

$$\sum_{i=0}^N \sum_{j=0, j \neq i}^N d_i x_{ijk rh} \leq q \quad \forall h \in H, k \in K, r \in R \quad (7)$$

$$\sum_{i=0}^N \sum_{j=0, j \neq i}^N x_{ijk rh} (t_{ij} + s_i) \leq Jk \quad \forall h \in H, k \in K, r \in R \quad (8)$$

$$w_{ikrh} + s_i + t_{ij} - M(1 - x_{ijkrh}) \leq w_{jkrh} \quad \forall i, j \in N, k \in K, r \in R, h \in H \quad (9)$$

$$w_{ikrh} + s_i + t_{i0} - M(1 - x_{i0krh}) \leq tback_r \quad \forall i \in N, k \in K, r \in R, h \in H \quad (10)$$

$$tstart_r + s_i + t_{0i} - M(1 - x_{0ikrh}) \leq w_{ikrh} \quad \forall i \in N, k \in K, r \in R, h \in H \quad (11)$$

$$\sum_{k=1}^K \sum_{j=1}^N x_{0jkrh} - \sum_{k=1}^K \sigma_{krh} = 0 \quad \forall h \in H, r \in R \quad (12)$$

$$\sigma_{krh} + \sigma_{kuh} - z_{kruh} - z_{kurh} \leq 1 \quad \forall k \in K, r, u \in R, h \in H \quad (13)$$

$$1 - z_{kruh} - z_{kurh} \geq 0 \quad \forall k \in K, r, u \in R, h \in H \quad (14)$$

$$tback_r + tstart_u \leq M(1 - z_{kruh}) \quad \forall k \in K, r, u \in R, h \in H \quad (15)$$

$$a_i y_{ikrh} \leq w_{ikrh} \leq b_i y_{ikrh} \quad \forall i \in N, k \in K, r \in R, h \in H \quad (16)$$

$$x_{ijkrh} \in \{0,1\} \quad \forall i, j \in N, k \in K, r \in R, h \in H \quad (17)$$

$$y_{ikrh} \in \{0,1\} \quad \forall i \in N, k \in K, r \in R, h \in H \quad (18)$$

$$z_{kruh} \in \{0,1\} \quad \forall k \in K, r, u \in R, h \in H \quad (19)$$

$$\sigma_{krh} \in \{0,1\} \quad \forall k \in K, r \in R, h \in H \quad (20)$$

Constraint (2) ensures that each route of each vehicle departs from the depot at most once on a given day. Constraints (3)–(5) maintain flow conservation at each customer and link the arc variables to the visit variables, and constraint (6) ensures that each customer is visited exactly once over all vehicles, routes, and days. Constraint (7) limits the load of each route to the vehicle capacity, and constraint (8) limits each route to the maximum delivery time. Constraints (9)–(11) ensure that customer j is visited only after service at the preceding customer i has been completed and that the arrival times are consistent with the start and return times of each route. Constraints (12)–(15) maintain route continuity by linking route usage to depot departures and by sequencing the routes performed by the same vehicle on the same day. Constraint (16) enforces the hard time windows, and constraints (17)–(20) define the binary decision variables.

Validation and Verification Procedure

Before the model was applied, its validity and internal consistency were examined. Validation assessed whether the model is able to produce correct and accurate solutions for the problem it is intended to represent [30]. Each model component, namely the decision variables, the objective function, and the constraints, was compared with the delivery activities of the distributor to confirm that every activity is represented by the formulation. Verification was then performed by unit verification, or dimensional analysis, in which the units of the left- and right-hand sides of the objective function and of each constraint were compared to confirm that the formulation is dimensionally consistent.

Test Scenarios and Computational Setup

Logical testing examined whether the model generates solutions that satisfy three conditions: (1) the model can generate one route in one day; (2) it can generate more than one route in one day; and (3) it can generate routes on different days. Three scenarios with different numbers of nodes and customer demands were constructed so that each condition could be examined in turn. The test data were taken from a distributor in Padang City that supplies snacks to supermarkets and whose delivery system matches the structure of the model, but the scenarios were deliberately made smaller than the actual problem. In all scenarios, the fleet consisted of two vehicles, each with a capacity of 50 bales, and the planning horizon was two days. Because the model contains a large number of variables, the scenarios were solved with LINGO software. Table 2 summarizes the settings of the three scenarios; the total demand of

each scenario, obtained by summing the customer demands, is included because it determines the minimum number of vehicle routes needed under the 50-bale capacity.

Table 2. Settings of the three test scenarios

Setting	Scenario 1	Scenario 2	Scenario 3
Number of customers	5	8	10
Number of nodes (including the depot)	6	9	11
Planning horizon (days)	2	2	2
Fleet	2 vehicles × 50 bales	2 vehicles × 50 bales	2 vehicles × 50 bales
Demand per customer (bales)	10 (customers 1–5)	15 (customers 1–8)	15 (customers 1–5); 20 (customers 6–10)
Total demand (bales)	50	120	175
Condition examined	(1)	(2)	(3)

Note: Total demand is the sum of the customer demands. Node 0 is the depot.

RESULTS AND DISCUSSION

Model Validation

The validation showed that each component of the formulation corresponds to an activity in the distributor's delivery process. Every activity is represented by at least one constraint, and the route index r was introduced specifically to represent the distributor's practice of dispatching the same vehicle on several trips within a day. The objective function, which minimizes transportation cost, corresponds directly to the distributor's aim of reducing the transportation costs it incurs. On this basis, the model was judged conceptually valid for representing the distributor's system; whether its solutions are logically correct was examined subsequently through the scenario tests.

Unit Verification

Conceptual validity does not guarantee that the functions are dimensionally consistent, so the units of every function were checked next. Table 3 reports the units of the left- and right-hand sides of the objective function and constraints (1)–(16); the binary domain constraints (17)–(20) are unitless by definition and are therefore not listed.

Table 3. Unit verification of the objective function and constraints

Equation	Function	Left-hand side unit	Right-hand side unit
(1)	Total transportation cost (objective)	IDR	IDR
(2)	Depot departure of each route	Unitless	Unitless
(3)	Flow conservation at each customer	Unitless	Unitless
(4)	Outgoing arcs linked to customer visits	Unitless	Unitless
(5)	Incoming arcs linked to customer visits	Unitless	Unitless
(6)	Single visit to each customer	Unitless	Unitless
(7)	Vehicle capacity	Capacity unit	Capacity unit
(8)	Maximum delivery time	Minutes	Minutes
(9)	Arrival-time sequence between customers	Minutes	Minutes
(10)	Return time of each route	Minutes	Minutes
(11)	Start time of each route	Minutes	Minutes
(12)	Route usage linked to depot departures	Unitless	Unitless
(13)	Ordering of routes of the same vehicle	Unitless	Unitless
(14)	Single precedence between two routes	Unitless	Unitless
(15)	Timing between successive routes	Minutes	Minutes
(16)	Hard time windows	Minutes	Minutes

Note: IDR = Indonesian rupiah. Equation numbers refer to the formulation in the Method section; the binary domain constraints (17)–(20) are unitless.

Both sides of every function carry the same unit. The objective function is expressed in Indonesian rupiah (IDR), the capacity constraint (7) in capacity units, the timing constraints (8)–(11), (15), and (16) in minutes, and the remaining flow, assignment, and sequencing constraints are unitless. The formulation is therefore dimensionally consistent and can be used to generate solutions.

Logical Testing of the Three Scenarios

The three scenarios were solved in order of increasing size, each targeting one of the conditions defined in the Method. [Table 4](#) lists the routes generated in each scenario together with the load of each route, computed from the customer demands in [Table 2](#), and [Figure 2](#) shows how these routes are distributed across vehicles, trips, and days.

Table 4. Routes generated by the MTMPVRPTW model in the three scenarios and their vehicle loads

Scenario	Day	Vehicle	Route	Node sequence	Load (bales)	Load factor (%)
1	Single day	1	1	0-3-1-2-5-4-0	50	100
2	Single day	1	1	0-7-6-3-0	45	90
		1	2	0-4-5-0	30	60
		2	1	0-8-2-1-0	45	90
3	1	1	–	0-3-7-5-0	50	100
	1	2	–	0-1-2-6-0	50	100
	2	1	–	0-4-10-0	35	70
	2	2	–	0-8-9-0	40	80

Note: Node 0 is the depot. Load is the sum of the demands of the customers on the route; load factor = $\text{load}/50 \times 100$. In Scenarios 1 and 2, all routes were performed on one of the two available days; in Scenario 3, route labels are reported by day and vehicle.

(a) Scenario 1 (one operating day)

Trip 1: 0-3-1-2-5-4-0

Vehicle 1 **50/50 bales**

Vehicle 2 *no route assigned*

(b) Scenario 2 (all routes on the same day)

Trip 1: 0-7-6-3-0

Trip 2: 0-4-5-0

Vehicle 1 **45/50 bales**

30/50 bales

Trip 1: 0-8-2-1-0

Vehicle 2 **45/50 bales**

(c) Scenario 3 (two operating days)

Trip 1: 0-3-7-5-0

Trip 1: 0-4-10-0

Vehicle 1 **50/50 bales**

35/50 bales

Trip 1: 0-1-2-6-0

Trip 1: 0-8-9-0

Vehicle 2 **50/50 bales**

40/50 bales

Day 1

Day 2

Figure 2. Route structure of the solutions generated in (a) Scenario 1, (b) Scenario 2, and (c) Scenario 3. Each bar represents one trip; the outlined length corresponds to the 50-bale vehicle capacity and the filled part to the load carried. Node 0 is the depot.

Scenario 1 examined the first condition with five customers, each demanding 10 bales. The model served all customers with a single route of vehicle 1 (0-3-1-2-5-4-0), and the solution was obtained in 2 s. Although two days were available, all customers were visited in a single

day (Figure 2a). Because the total demand of 50 bales equals the capacity of one vehicle, this route operated at a load factor of 100%. Condition (1) was therefore satisfied.

Scenario 2 enlarged the instance to eight customers with a demand of 15 bales each. The total demand of 120 bales exceeds the capacity of two single-trip vehicles (100 bales), and the model generated three routes on the same day: vehicle 1 performed two trips (0–7–6–3–0 and 0–4–5–0), and vehicle 2 performed one trip (0–8–2–1–0), with loads of 45, 30, and 45 bales, respectively (Table 4). Because vehicle 1 returned to the depot and was dispatched again on the same day (Figure 2b), the solution contains a multi-trip operation, and conditions (1) and (2) were both satisfied. This scenario required more computation time than Scenario 1 because it involved more nodes.

Scenario 3 examined the third condition with ten customers whose demands were 15 bales (customers 1–5) or 20 bales (customers 6–10), giving a total demand of 175 bales. The model generated four routes over two days, with each vehicle performing one route per day (Figure 2c). On day 1, vehicle 1 served 0–3–7–5–0 and vehicle 2 served 0–1–2–6–0, both at the full load of 50 bales; on day 2, vehicle 1 served 0–4–10–0 (35 bales) and vehicle 2 served 0–8–9–0 (40 bales). Deliveries therefore extended over more than one day, demonstrating a multi-period operation and satisfying condition (3).

Computational Performance

Beyond satisfying the three conditions, the scenarios also indicate how efficiently the solutions use the fleet and how the computation time develops with instance size. Table 5 summarizes these outcomes.

Table 5. Fleet use and computation time of the three scenarios

Scenario	Nodes	Routes generated	Capacity-based minimum routes	Mean load factor (%)	Running time	Conditions satisfied
1	6	1	1	100.0	2 s	(1)
2	9	3	3	80.0	3 min 26 s (206 s)	(1), (2)
3	11	4	4	87.5	30 min 11 s (1,811 s)	(3)

Note: Capacity-based minimum routes = $\lceil \text{total demand}/q \rceil$ with $q = 50$ bales; mean load factor = $\text{total demand}/(\text{routes} \times q) \times 100$. Running times are those reported by LINGO.

In every scenario, the number of generated routes equaled the capacity-based minimum $\lceil \sum d_i/q \rceil$, that is, one, three, and four routes, so no scenario used more routes than its total demand required, and the mean load factor ranged from 80.0% to 100%. The running time, however, increased sharply with instance size: from 2 s for 6 nodes to 3 min 26 s (206 s) for 9 nodes and 30 min 11 s (1,811 s) for 11 nodes. Adding three nodes from Scenario 1 to Scenario 2 increased the running time approximately 100-fold, and adding two further nodes increased it by a further factor of about 8.8.

Discussion

Taken together, the scenario tests (Table 4, Figure 2) show that the formulation reproduces the three structural behaviors required of an MTMPVRPTW: a single-trip solution when one route suffices, a second trip by the same vehicle within one day, and the allocation of routes to different days. The second behavior depends on the route index r and the precedence variables Z_{kruh} , which allow a vehicle to return to the depot and start a new route on the same day, as vehicle 1 did in Scenario 2. This combination distinguishes the model from its two sources. The formulation of Gökçe and Ercan [26] spreads the routes of heterogeneous vehicles over several days and allows repeated visits, whereas the MTPVRPTW-LD [27] sequences multiple trips of a

single vehicle within a limited working duration. The present model retains the multi-day horizon of the former and the trip sequencing of the latter while restricting each customer to a single visit by a homogeneous fleet, which is the configuration that motivated this study.

The way the model distributed routes in Scenario 3 points to the role of the time-related constraints. The 175-bale demand required at least four full or partial vehicle loads, which could in principle have been delivered as two trips per vehicle on a single day, as in Scenario 2; instead, the model assigned one route per vehicle on each of two days. Because the objective function contains transportation cost only and no cost for using an additional day, this allocation is governed by the hard time windows (16) and the maximum delivery time (8) rather than by the objective itself. Under hard windows, a vehicle that would arrive late cannot simply absorb a penalty, as it could in a soft-window formulation [7], and an early arrival produces waiting time [8]; a second trip on the same day is therefore feasible only when the remaining customers can still be reached within their windows. The time-window and travel-time parameters of the test instances are not analyzed here, so this explanation remains a plausible interpretation rather than a demonstrated mechanism.

The tests also show that the solutions used the fleet capacity efficiently (Table 5). The number of routes matched the capacity-based minimum in every scenario, and two of the four routes in Scenario 3 and the single route in Scenario 1 were fully loaded. This result is relevant to the distributor's objective because each additional route adds travel from and back to the depot. Route-planning studies of comparable Indonesian distribution and transportation cases have used the savings matrix method to reduce distribution costs [3], the Clarke–Wright savings algorithm to shorten waste-collection routes [28], and cuckoo search to determine food-assistance distribution routes [29]. These methods construct efficient routes quickly, whereas the MTMPVRPTW explicitly decides on which trip and on which day each route is performed and guarantees that no time window is violated. The two approaches are therefore complementary rather than competing.

The price of this explicit representation is computational. The roughly 100-fold increase in running time between Scenarios 1 and 2 and the further increase to more than 30 min in Scenario 3 (Table 5) are consistent with the NP-hardness of the MTPVRPTW [10], [11], in which the number of possible assignments of customers to vehicles, trips, and days grows combinatorially. Dedicated exact algorithms have been developed for multi-trip routing with time windows [14], [27], whereas larger multi-trip and multi-period instances have generally been solved with heuristic or metaheuristic methods, such as adaptive large neighborhood search [17], hybrid estimation-of-distribution and evolutionary algorithms [8], [9], and tabu search combined with variable neighborhood search [11], [19]. For the distributor's full-scale problem, the present model is therefore best used as a reference formulation: its exact solutions on small instances can serve as benchmarks for validating faster heuristic solvers, in line with standardized procedures for validating vehicle routing algorithms [30].

Several limitations bound these conclusions. First, the tests used three reduced instances with at most ten customers and two vehicles, so the results establish the logical behavior of the model rather than its performance at the scale of the actual distribution problem. Second, the analysis focuses on route structure; the transportation-cost values of the solutions and the time-window, travel-time, and service-time parameters of the instances are not analyzed, so neither the cost savings relative to current practice nor the binding constraints can be quantified. Third, the validation was conceptual and was not accompanied by a comparison with the distributor's historical routes. Applying the model to full-scale data, comparing its solutions with existing routes, and developing heuristic solution methods for larger instances are therefore the necessary next steps before the model can support operational decisions.

CONCLUSION

This study formulated a mixed-integer linear programming model for the multi-trip multi-period vehicle routing problem with hard time windows by integrating multi-trip and multi-period VRPTW formulations for a distributor that operates a homogeneous fleet, serves each customer once, and plans an unrestricted number of daily trips over several days. The model was judged conceptually consistent with the distributor's delivery operations and was dimensionally consistent in all its functions. In tests on three reduced instances, it generated a single route in one day, a second trip by the same vehicle within one day, and routes spread over two days, with every route within vehicle capacity and the number of routes equal to the capacity-based minimum. These results indicate that the formulation behaves as intended for single-trip, multi-trip, and multi-period routing under hard time windows. However, solution time increased from seconds to about half an hour as the instance grew from five to ten customers, so the model in its present form is suited to small instances and to benchmarking faster solution methods. Full-scale application with actual cost and time-window data, comparison with the distributor's current routes, and the development of heuristic solution approaches are required before operational cost savings can be claimed.

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